

CHAPTER TWENTY THREE

CMOS Inverter

Introduction

Complementary MOS (CMOS) uses a P-channel MOS as a pull-up load device with an N-channel MOS as a pull-down device.

CMOS is widely used in digital circuit technology because it possesses the lowest power dissipation (wrist watches, hand-held calculators, and recently note book computers) and has the highest packing density.

All modern microprocessors are manufactured using CMOS technology including Intel's 80286, 80386, 80486, 8086, 8088 and Motorola's 68010, 68020, 68030, 68040

In this chapter, we will describe the operation of CMOS as an inverter device, and then analyze its VTC, power-dissipation, and fan-out.

Operation of CMOS Inverter

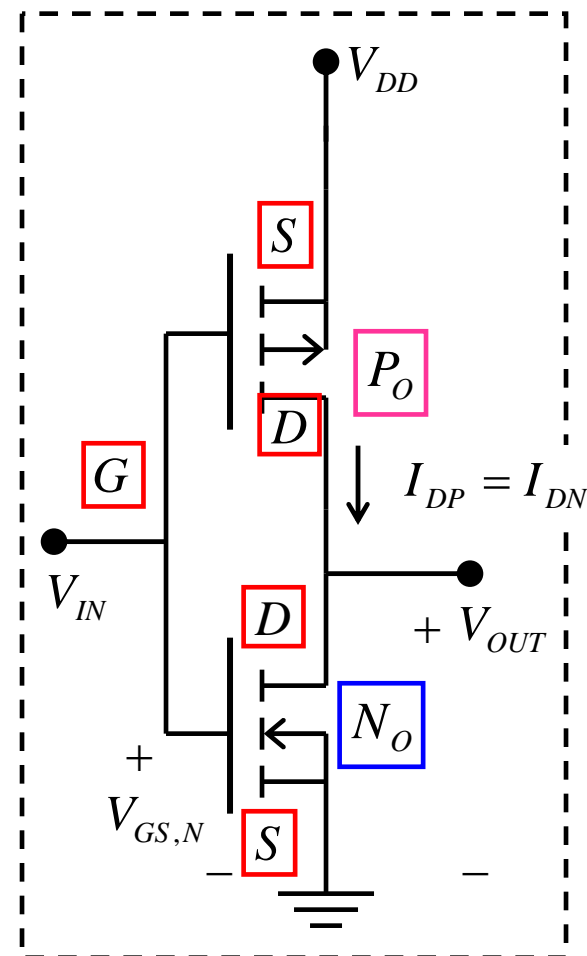
The drain terminal of the NMOS is connected with the drain terminal of the PMOS

$$V_{IN} = V_{GS,N} = V_{DD} - V_{SG,P}$$

The output is taken at the common drain terminal

$$V_{OUT} = V_{DS,N} = V_{DD} - V_{SD,P}$$

Note: either MOS can be considered as a load for the other, therefore the operations of N_O and P_O complement each other.



VTC of CMOS Inverter

V_{OH}

When $V_{IN}=0$, i.e. $V_{IN}=V_{GS,N} < V_{TN}$: $\rightarrow N_O$ is cut-off

$$I_{D,N} = I_{D,P} = 0$$

$$\underline{P_O} \quad V_{SG,P} = V_{DD} - V_{IN} = V_{DD}$$

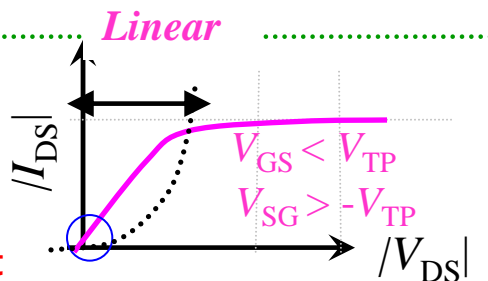
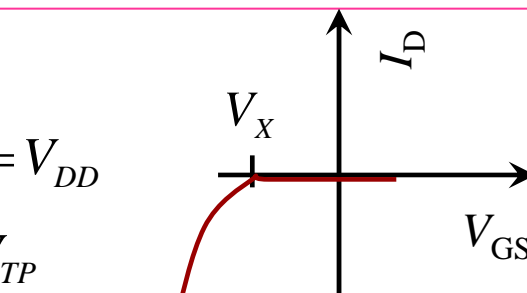
$$V_{GS,P} = -V_{DD} < V_{TP}$$

\rightarrow Channel is created between the source and the drain of P_O MOS

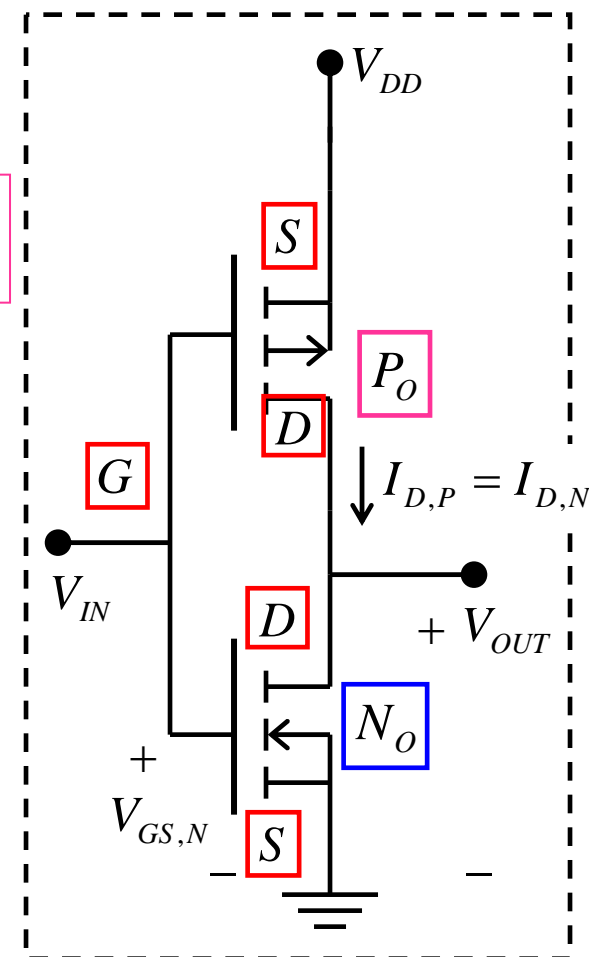
In active mode: ($V_{GS} \leq V_{TP}$)

Zero drain current

Slide 3 of chapter 17



$\rightarrow P_O$ is active (linear) mode



VTC of CMOS Inverter

V_{OH}

When $V_{IN}=0$, i.e. $V_{IN}=V_{GS,N} < V_{TN}$: $\rightarrow N_O$ is cut-off

$$I_{D,N} = I_{D,P} = 0$$

$\rightarrow P_O$ is active (linear) mode

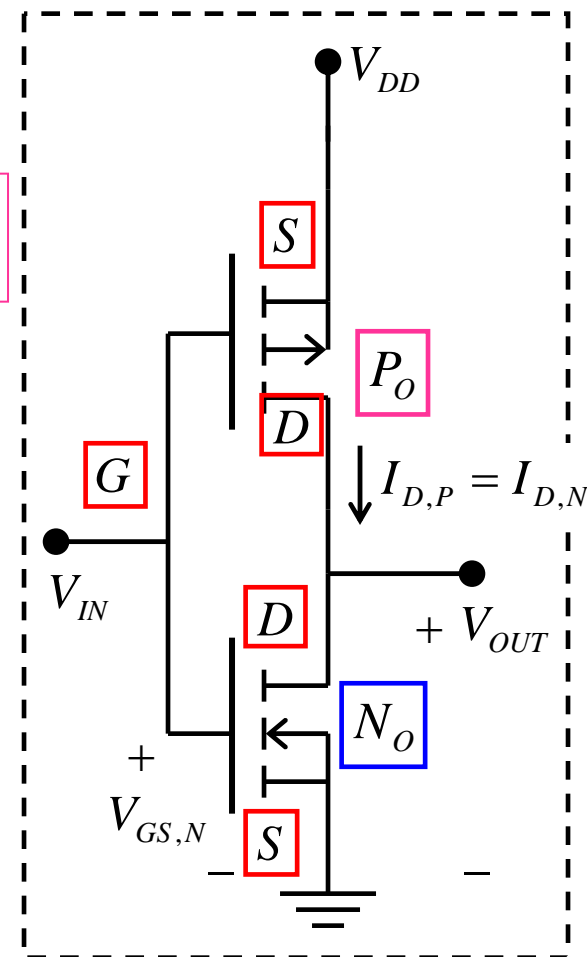
$$I_{SD,P}(LIN) = \frac{K_p}{2} \left[2 \times (V_{SG,P} + V_{TP}) V_{SD,P} - V_{SD,P}^2 \right] = 0$$

$$V_{SD,P} \{ 2 \times (V_{SG,P} + V_{TP}) - V_{SD,P} \} = 0$$

$$V_{SD,P} = 0 \quad \checkmark$$

$$V_{SD,P} = 2 \times (V_{SG,P} + V_{TP}) > V_{SG,P} + V_{TP} \quad \times$$

$$V_{OH} = V_{DS,N} = V_{DD} - V_{SD,P} = V_{DD}$$



Invalid since V_{SD} has to be less than $(V_{SG} + V_{TP})$

VTC of CMOS Inverter

V_{OL}

For $V_{IN} = V_{OH} = V_{DD}$ ($V_{SG,P} = 0 > V_{T,P}^{(-)}$)

→ P_O is cut-off

Active mode: ($V_{GS} \geq V_{TN}$) **Zero drain current**

→ N_O is active, linear

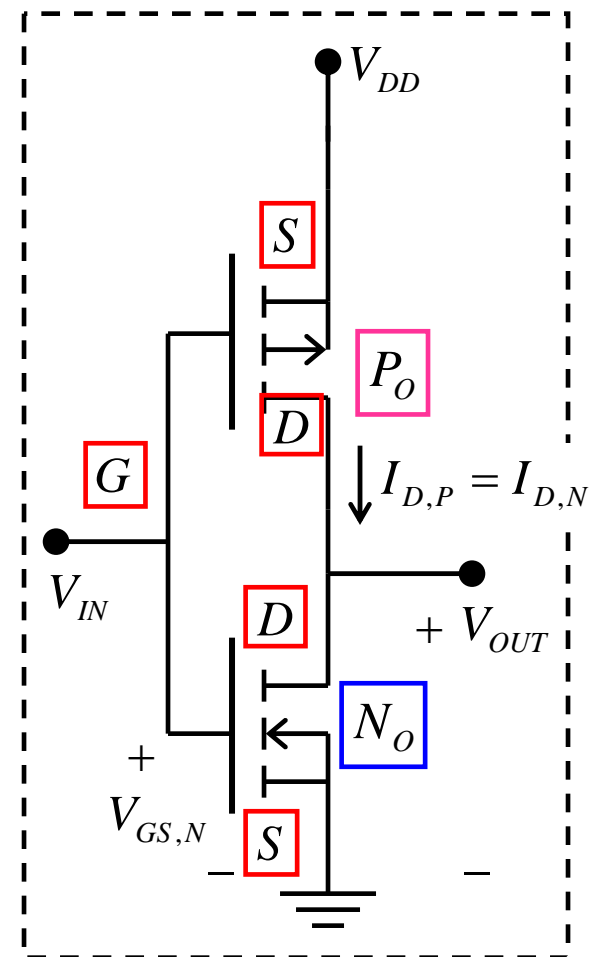
$$I_{DS,N}(LIN) = \frac{K_n}{2} [2 \times (V_{GS,N} - V_{TN}) V_{DS,N} - V_{DS,N}^2] = 0$$

$$V_{DS,N} \{2 \times (V_{GS,N} - V_{TN}) - V_{DS,N}\} = 0$$

$$V_{DS,N} = 0 \quad \checkmark$$

$$V_{DS,N} = 2 \times (V_{GS,N} - V_{TN}) > V_{GS,N} - V_{TN} \quad \times$$

$$V_{OL} = V_{DS,N} = 0$$



Invalid since V_{DS} has to be less than $(V_{GS} - V_{TN})$

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Ch. 23

VTC of CMOS Inverter

N_O is cut-off & P_O is active, linear

for $V_O = V_{OH}$ ($V_I < V_{IL}$) \rightarrow N_O is cut-off & P_O is linear

for $V_I = V_{IL}$ \rightarrow N_O is sat & P_O is linear

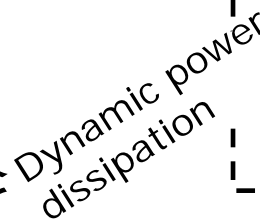
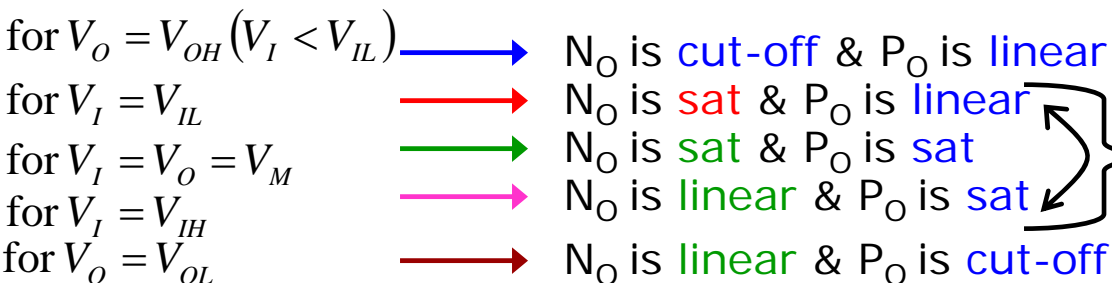
for $V_I = V_O = V_M$ \rightarrow N_O is sat & P_O is sat

for $V_I = V_{IH}$ \rightarrow N_O is linear & P_O is sat


for $V_O = V_{OL}$ \rightarrow N_O is linear & P_O is cut-off


Dynamic power dissipation

7



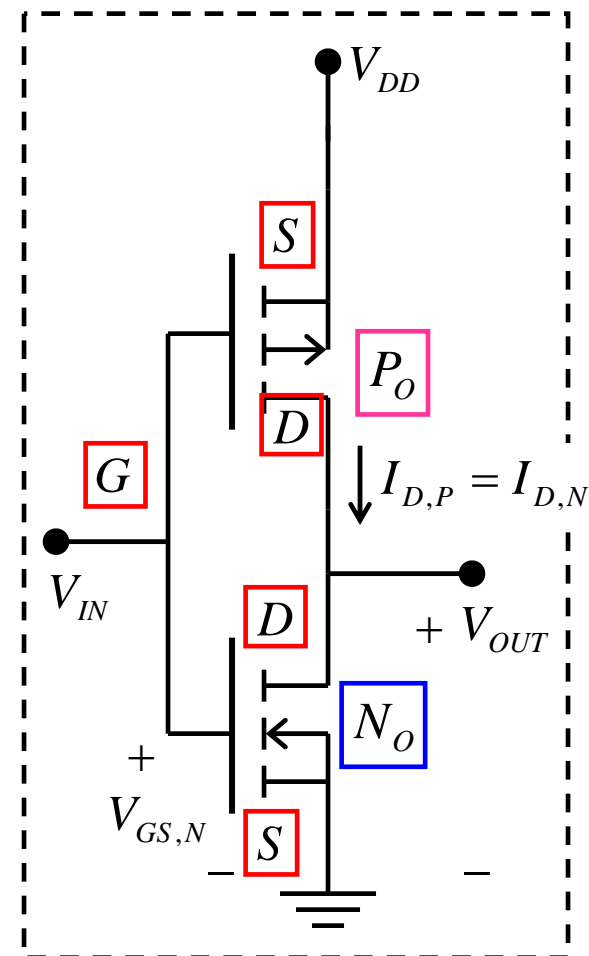
Power Dissipation CMOS Inverter

$I_{DD}(OL)$  $\rightarrow P_O$ is cut-off

$I_{DD}(OH)$  $\rightarrow N_O$ is cut-off

Static Power Dissipation $P_{DD}(avg)$

$$P_{DD}(avg) = V_{DD} \left(\frac{I_{DD}(OL) + I_{DD}(OH)}{2} \right) = 0$$



Power Dissipation CMOS Inverter

Dynamic Power Dissipation $P_{DD}(\text{dyn})$

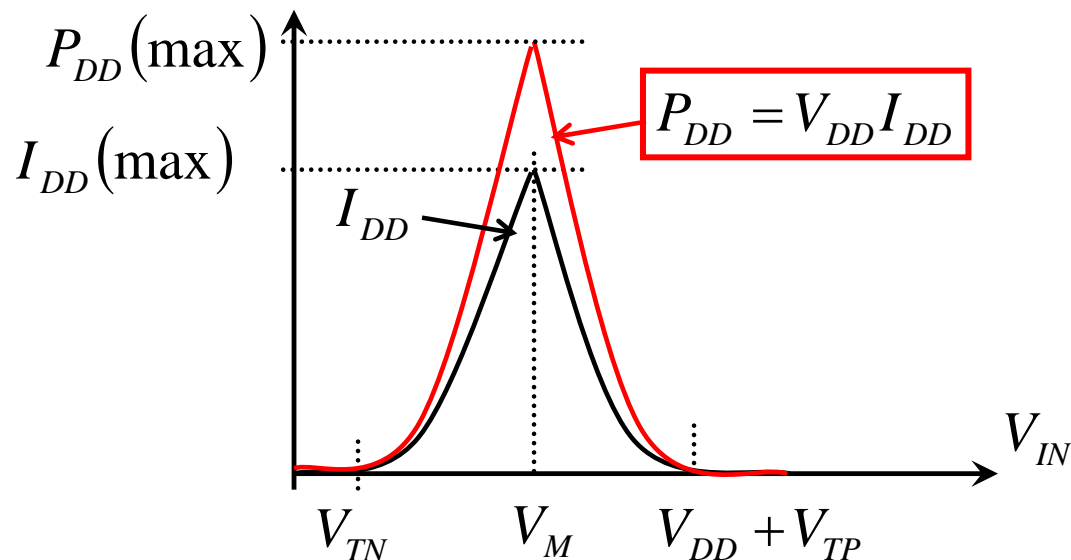
$$P_{DD}(\text{dyn}) = C_L \nu (V_{DD})^2$$

C_L is the total load capacitance at the output of the gate

ν is the switching frequency of the gate

$$P_{DD}(\text{CMOS}) = P_{DD}(\text{dyn})$$

See example 23.1



VTC of CMOS Inverter

Analytical determination of VTC :

$$V_{OH}$$

for $V_O = V_{OH} (V_I < V_{IL}) \rightarrow$

N_O is cut-off & P_O is linear

$$I_{D,N} = I_{D,P} = 0$$

$$V_{OH} = V_{DD}$$

$$V_{DS,N} = V_{DD}$$

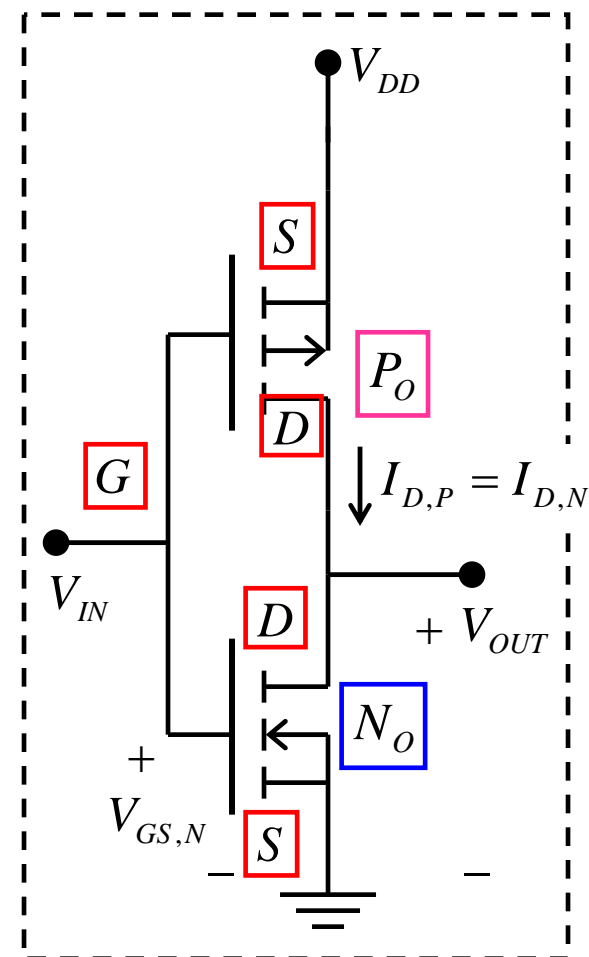
$$V_{OL}$$

for $V_O = V_{OL} \rightarrow$ N_O is linear & P_O is cut-off

$$I_{D,N} = I_{D,P} = 0$$

$$V_{OL} = 0$$

$$V_{DS,N} = 0$$



VTC of CMOS Inverter

Analytical determination of VTC :

V_{IL} for $V_I = V_{IL} \longrightarrow N_O$ is **sat** & P_O is **linear**

$$\begin{array}{ll} V_{GS} = V_{IL} & V_{SG,P} = V_{DD} - V_{IL} \\ V_{DS,N} = V_{Out} & V_{SD,P} = V_{DD} - V_{Out} \end{array}$$

$$I_{D,N} = \frac{K_n}{2} (V_{GS} - V_{TN})^2 \Rightarrow I_{D,N} = \frac{K_n}{2} (V_{IN} - V_{TN})^2$$

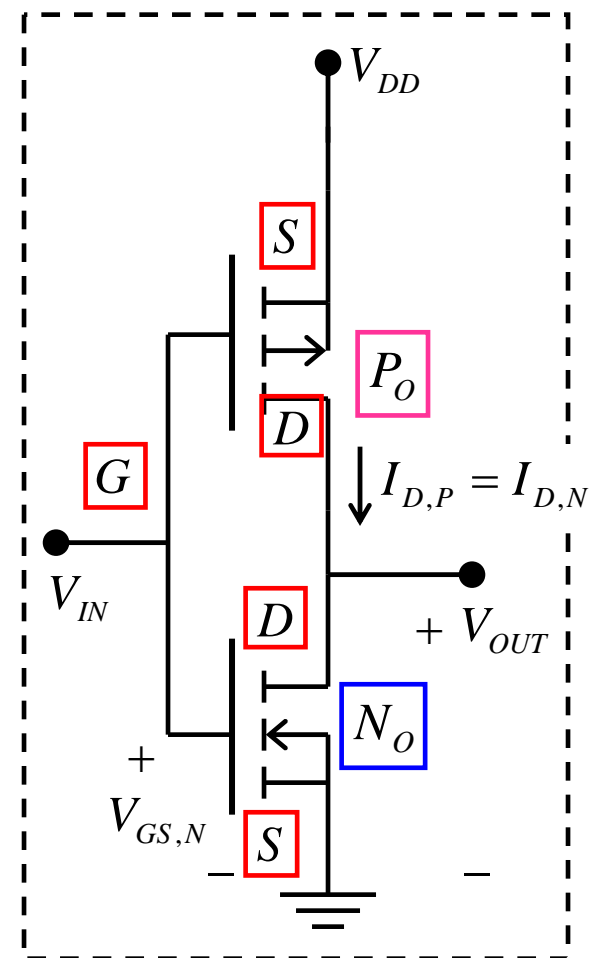
$$I_{D,P} = \frac{K_p}{2} [2 \times (V_{SG} + V_{TP}) V_{SD} - V_{SD}^2]$$

$$I_{D,P} = \frac{K_p}{2} [2 \times (V_{DD} - V_{IN} + V_{TP})(V_{DD} - V_{OUT}) - (V_{DD} - V_{OUT})^2]$$

$$I_{D,N} = I_{D,P}$$

or

$$\frac{\partial I_{D,N}}{\partial V_{IN}} = \frac{\partial I_{D,P}}{\partial V_{IN}}$$



VTC of CMOS Inverter

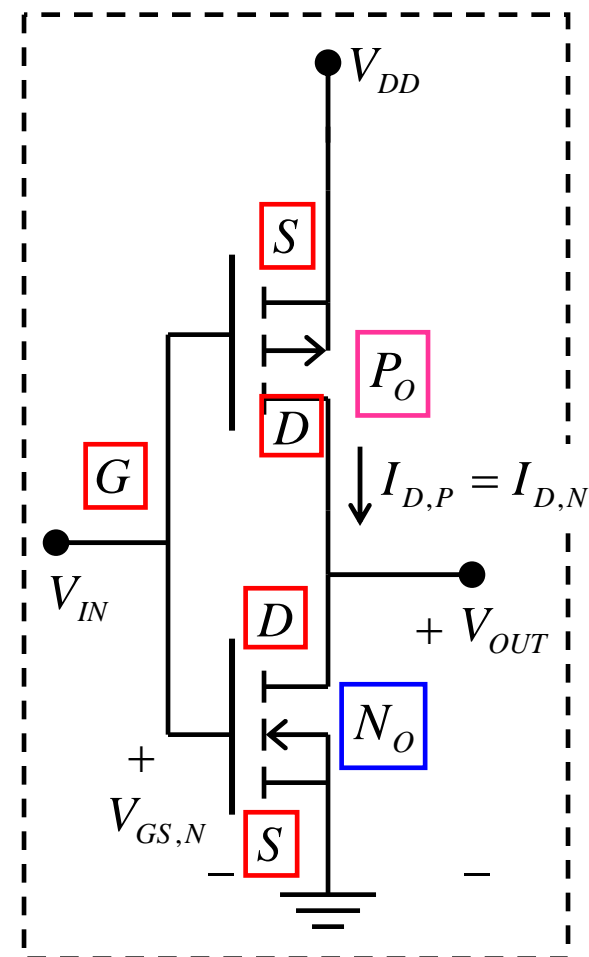
Analytical determination of VTC :

V_{IL} for $V_I = V_{IL} \longrightarrow N_O$ is **sat** & P_O is **linear**

$$\begin{array}{ll} V_{GS} = V_{IL} & V_{SG,P} = V_{DD} - V_{IL} \\ V_{DS,N} = V_{Out} & V_{SD,P} = V_{DD} - V_{Out} \end{array}$$

$$\frac{dV_{OUT}}{dV_{IN}} \bigg|_{V_{IN}=V_{IL}} = -1$$

$$V_{IL} = \frac{2K_p V_{OUT} - K_p (V_{DD} - V_{TP}) + K_n V_{TN}}{K_n + K_p}$$



Analytical determination of VTC :

$$\begin{array}{ll} V_{GS} = V_{IH} & V_{SG,P} = V_{DD} - V_{IH} \\ V_{DS,N} = V_{Out} & V_{SD,P} = V_{DD} - V_{Out} \end{array}$$

$$I_{D,p} = \frac{K_p}{2} (V_{DD} - V_{IN} + V_{TP})^2$$

or

$$\frac{\partial I_{D,N}}{\partial V_{IN}} = \frac{\partial I_{D,P}}{\partial V_{IN}}$$



VTC of CMOS Inverter

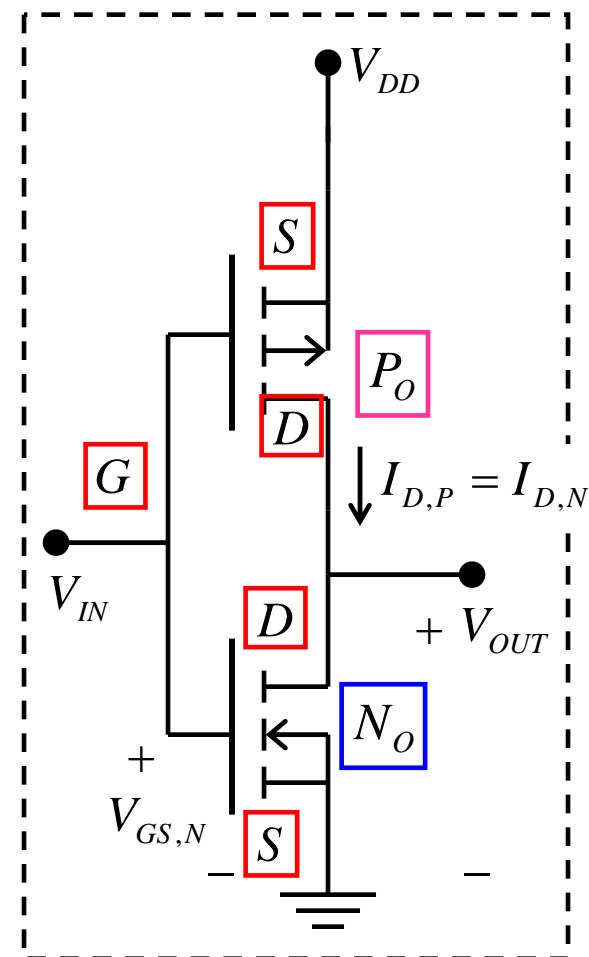
Analytical determination of VTC :

V_{IH} for $V_I = V_{IH} \longrightarrow$ N_O is linear & P_O is sat

$$\begin{array}{ll} V_{GS} = V_{IH} & V_{SG,P} = V_{DD} - V_{IH} \\ V_{DS,N} = V_{Out} & V_{SD,P} = V_{DD} - V_{Out} \end{array}$$

$$\frac{dV_{OUT}}{dV_{IN}} \bigg|_{V_{IN}=V_{IH}} = -1$$

$$V_{IH} = \frac{2K_n V_{OUT} + K_p (V_{DD} + V_{TP}) + K_n V_{TN}}{K_n + K_p}$$



VTC of CMOS Inverter

Analytical determination of VTC :

V_M Mid point

for $V_I = V_O = V_M$

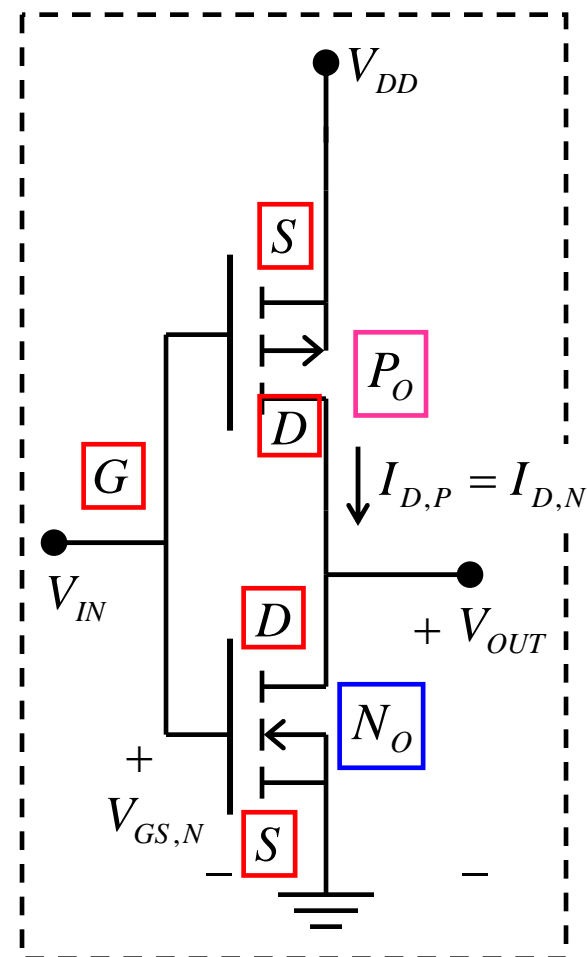


N_O is sat & P_O is sat

$$\frac{K_n}{2} (V_M - V_{TN})^2 = \frac{K_p}{2} ((V_{DD} - V_M) + V_{TP})^2$$

Then solve for V_M

See example on page 342



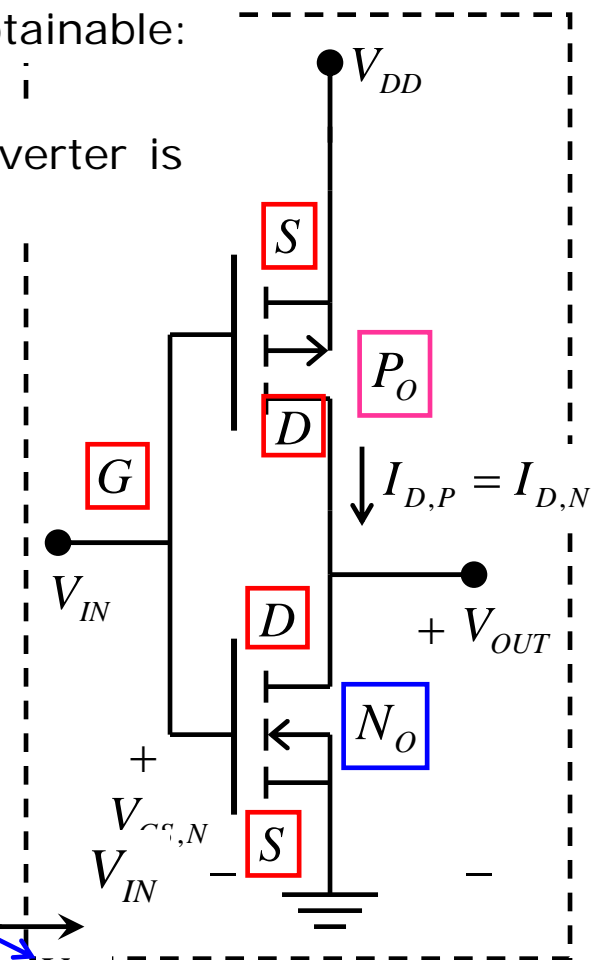
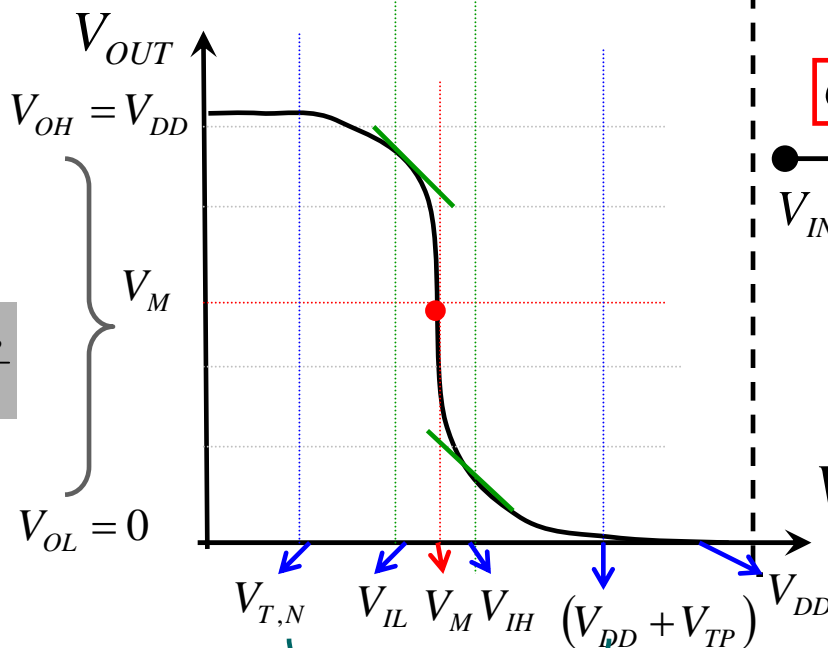
The Symmetric CMOS Inverter

- The VTC of symmetric CMOS inverter is easily obtainable:
- One reason of designing a symmetric CMOS inverter is to obtain a symmetric transient response:
- ❖ Remember the VTC of CMOS inverter

$$V_M = \frac{V_{OH} + V_{OL}}{2} = \frac{V_{DD}}{2}$$

$$V_M = \frac{V_{IH} + V_{IL}}{2} = \frac{V_{TN} + V_{DD} + V_{TP}}{2}$$

$$V_{TN} = -V_{TP}$$



Design of Symmetric CMOS Inverter

1. The threshold voltages of N-MOS and P-MOS are made equal in magnitude
2. The requirements for $K_n = K_p$ must be considered

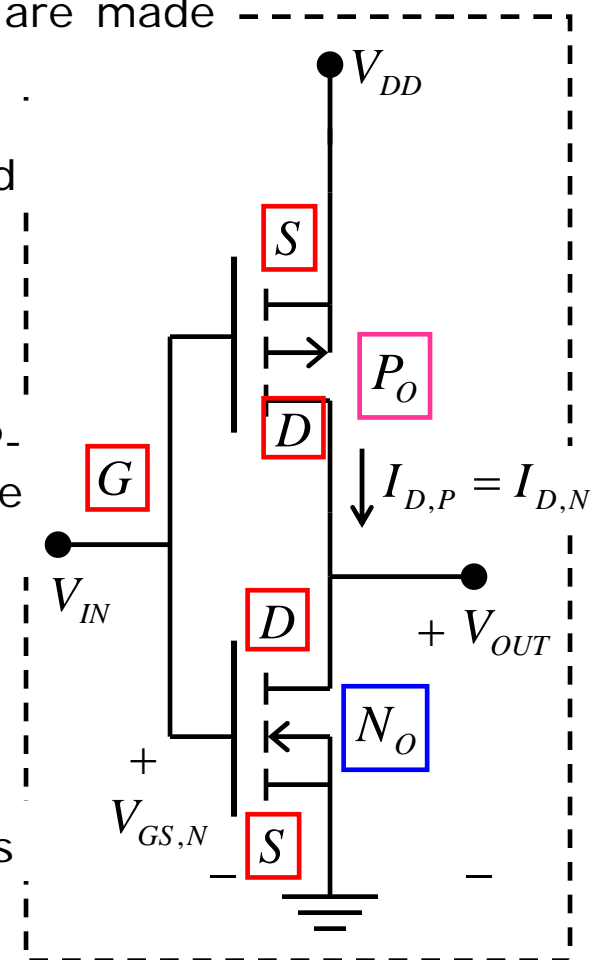
$$K_n = \frac{W_n}{L_n} \mu_n C_{ox} \quad K_p = \frac{W_p}{L_p} \mu_p C_{ox}$$

3. Usually the gate oxide layers of the N-MOS and P-MOS devices are grown simultaneously, and hence have the same thickness t_{ox}

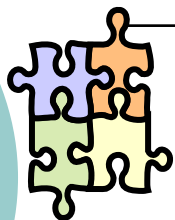
$$\frac{W_n}{L_n} \mu_n = \frac{W_p}{L_p} \mu_p$$

4. Typically $\mu_n(\text{Si}) = 580 \text{ cm}^2/\text{V.s}$ and $\mu_p(\text{Si}) = 230 \text{ cm}^2/\text{V.s}$

$$\frac{W_p}{L_p} = 2.5 \frac{W_n}{L_n}$$



Design of Symmetric CMOS Inverter



○ Example

Design a symmetrical CMOS inverter assuming:

$$V_{DD}=5V, \quad V_{TN}=1V, \quad V_{TP}=-1V, \quad \mu_n C_{ox}=40 \mu A/V^2, \quad \mu_p C_{ox}=16 \mu A/V^2, \\ W_n=4 \mu m, \quad L_n=L_p=2 \mu m$$

And verify that the midpoint voltage is half of V_{DD} , and V_{IL} and V_{IH} are symmetric about V_M .

○ Solution

For symmetry, the width of the channel of P-MOS is determined from

$$\frac{W_p}{L_p} = 2.5 \frac{W_n}{L_n} \Rightarrow W_p = 2.5 \times 4 = 10 \mu m$$

$$K_n = \frac{W_n}{L_n} \mu_n C_{ox}$$

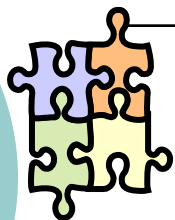
$$K_n = 2 \times 40 = 80 \mu A/V^2$$

$$K_p = \frac{W_p}{L_p} \mu_p C_{ox}$$

$$K_p = 5 \times 16 = 80 \mu A/V^2$$

Equal values

Design of Symmetric CMOS Inverter



○ Example

Design a symmetrical CMOS inverter assuming:

$$V_{DD}=5V, V_{TN}=1V, V_{TP}=-1V, \mu_n C_{ox}=40 \mu A/V^2, \mu_p C_{ox}=16 \mu A/V^2, \\ W_n=4 \mu m, L_n=L_p=2 \mu m$$

And verify that the midpoint voltage is half of V_{DD} , and V_{IL} and V_{IH} are symmetric about V_M .

○ Solution

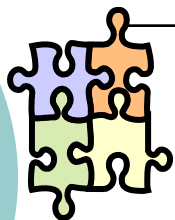
$$\frac{K_n}{2} (V_M - V_{TN})^2 = \frac{K_p}{2} ((V_{DD} - V_M) + V_{TP})^2$$

Slide 15 of this chapter

$$(V_M - 1)^2 = ((5 - V_M) - 1)^2$$

$$2V_M = 5 \Rightarrow V_M = 2.5V \Rightarrow V_M = \frac{V_{DD}}{2}$$

Design of Symmetric CMOS Inverter



Example

Design a symmetrical CMOS inverter assuming:

$$V_{DD}=5V, V_{TN}=1V, V_{TP}=-1V, \mu_n C_{ox}=40 \mu A/V^2, \mu_p C_{ox}=16 \mu A/V^2, \\ W_n=4 \mu m, L_n=L_p=2 \mu m$$

And verify that the midpoint voltage is half of V_{DD} , and V_{IL} and V_{IH} are symmetric about V_M .

Solution

Slide 12 of this chapter

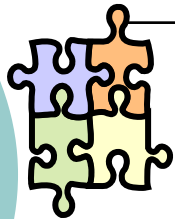
$$V_{IL} = \frac{2K_p V_{OUT} - K_p (V_{DD} - V_{TP}) + K_n V_{TN}}{K_n + K_p}$$



$$V_{IL} = \frac{160V_{OUT} - 80(5+1) + 80}{160} = V_{OUT} - 2.5$$

$$I_{D,N} = \frac{K_n}{2} (V_{IN} - V_{TN})^2 = I_{D,P} = \frac{K_p}{2} [2 \times (V_{SG} + V_{TP}) V_{SD} - V_{SD}^2] \\ \frac{K_n}{2} (V_{IL} - V_{TN})^2 = \frac{K_p}{2} [2 \times ((V_{DD} - V_{IL}) + V_{TP})(V_{DD} - V_{OUT}) - (V_{DD} - V_{OUT})^2] \\ (V_{IL} - 1)^2 = [2 \times ((5 - V_{IL}) - 1)(5 - V_{IL} - 2.5) - (5 - V_{IL} - 2.5)^2]$$

Design of Symmetric CMOS Inverter



○ Example

Design a symmetrical CMOS inverter assuming:

Ch. 23 $V_{DD}=5V$, $V_{TN}=1V$, $V_{TP}=-1V$, $\mu_n C_{ox}=40 \mu A/V^2$, $\mu_p C_{ox}=16 \mu A/V^2$,
 $W_n=4 \mu m$, $L_n=L_p=2 \mu m$

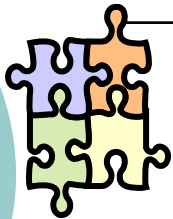
And verify that the midpoint voltage is half of V_{DD} , and V_{IL} and V_{IH} are symmetric about V_M .

○ Solution

$$(V_{IL} - 1)^2 = \left[2 \times ((5 - V_{IL}) - 1)(5 - V_{IL} - 2.5) - (5 - V_{IL} - 2.5)^2 \right]$$

$$(V_{IL} - 1)^2 = \left[2 \times (4 - V_{IL})(2.5 - V_{IL}) - (2.5 - V_{IL})^2 \right] \Rightarrow V_{IL} = \frac{12.75}{6} = 2.125V$$

Design of Symmetric CMOS Inverter



Example

Design a symmetrical CMOS inverter assuming:

Ch. 23 $V_{DD}=5V$, $V_{TN}=1V$, $V_{TP}=-1V$, $\mu_n C_{ox}=40 \mu A/V^2$, $\mu_p C_{ox}=16 \mu A/V^2$,
 $W_n=4 \mu m$, $L_n=L_p=2 \mu m$

And verify that the midpoint voltage is half of V_{DD} , and V_{IL} and V_{IH} are symmetric about V_M .

Solution : similarly

Slide 14 of this chapter

$$V_{IH} = \frac{2K_n V_{OUT} + K_p (V_{DD} + V_{TP}) + K_n V_{TN}}{K_n + K_p}$$



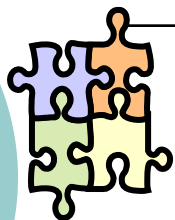
$$V_{IH} = \frac{160V_{OUT} + 80(5-1) + 80}{160} = V_{OUT} + 2.5$$

$$I_{D,N} = \frac{K_n}{2} (V_{IN} - V_{TN})^2 = I_{D,P} = \frac{K_p}{2} [2 \times (V_{SG} + V_{TP}) V_{SD} - V_{SD}^2]$$

$$\frac{K_n}{2} (V_{IH} - V_{TN})^2 = \frac{K_p}{2} [2 \times ((V_{DD} - V_{IH}) + V_{TP})(V_{DD} - V_{OUT}) - (V_{DD} - V_{OUT})^2]$$

$$(V_{IH} - 1)^2 = [2 \times ((5 - V_{IH}) - 1)(5 - V_{IH} + 2.5) - (5 - V_{IH} + 2.5)^2] \Rightarrow V_{IH} = 2.875V$$

Design of Symmetric CMOS Inverter



○ Example

Design a symmetrical CMOS inverter assuming:

$$V_{DD}=5V, \quad V_{TN}=1V, \quad V_{TP}=-1V, \quad \mu_n C_{ox}=40 \mu A/V^2, \quad \mu_p C_{ox}=16 \mu A/V^2, \\ W_n=4 \mu m, \quad L_n=L_p=2 \mu m$$

And verify that the midpoint voltage is half of V_{DD} , and V_{IL} and V_{IH} are symmetric about V_M .

○ Solution

$$\begin{array}{l} V_{IL} = 2.125V \\ V_{IH} = 2.875V \end{array} \Rightarrow \frac{V_{IH} + V_{IL}}{2} = \frac{2.875 + 2.125}{2} = 2.5V = V_M$$

V_{IL} and V_{IH} are symmetric about V_M .



-
- HW #12: Solve Problems: 23.1-3, 23.22